2025年度共同利用研究報告書

2026年03月04日

所属・職名 Kyushu University, Graduate School of Mathematics・PhD student

Fabbrini Edoardo

整理番号	2025a033

1.研究計画題目	Finite Element solver for kinematically incompatible non-simply connected Föppl-von Kármán plates			
2.新規・継続	継続			
3.種別	一般研究			
4.種目	短期共同研究			
5.開催方法	オンライン開催			
6.研究代表者	氏名	Fabbrini Edoardo		
	所属	Kyushu University, Graduate	職名	PhD student
7.研究実施期間	2025年04月21日(月曜日)～2025年05月03日(土曜日)			
	2026年02月24日(火曜日)～2026年02月26日(木曜日)			
8.キーワード	Multi-agent dynamics, mean field limits			
9.参加者人数	22人			

10.本研究で得られた成果の概要

This report summarizes the activities and outcomes of the joint research seminar "Multi-agent dynamics with labels and their continuum limit", held on February 24-26, 2026 within the framework of the IMI Joint Research Project 2025a033. The seminar consisted of three lectures delivered by Prof. Marco Morandotti (Politecnico di Torino), each lasting approximately 45 minutes and followed by discussion sessions with participants from several institutions.

The seminar introduced an analytical framework for the study of multi-agent systems in which the dynamics of agents depend not only on their spatial configuration but also on strategic or categorical information represented by labels. Such models arise in a wide range of applications, including collective behavior, evolutionary game theory, and social or biological dynamics. The lectures presented mathematical approaches for analyzing these systems and focused in particular on the passage from discrete agent-based models to continuum descriptions. In the limit of a large number of agents, the empirical distribution of agents can be described by partial differential equations such as continuity or Fokker-Planck equations.

The seminar provided participants with an overview of recent developments in the mathematical analysis of multi-agent systems and stimulated discussions on possible applications and further research directions.

Report on the Joint Research Project (2025a033)

Multi-agent dynamics with labels and their continuum limit

Principal Investigator: Dr. Edoardo Fabbrini[†]
Organizing Committee: Professor Pierluigi Cesana[*], Dr. Edoardo Fabbrini[†]
Lecturer: Professor Marco Morandotti[††]

[†]SACRA, Graduate School of Science, Kyoto University, Japan
[*]Institute of Mathematics for Industry (IMI), Kyushu University, Japan
[‡]Politecnico di Torino, Italy

Overview

This report summarizes the activities and outcomes of the joint research seminar "Multi-agent dynamics with labels and their continuum limit," held on February 24–26, 2026, within the framework of the IMI Joint Research Project 2025a033. The seminar consisted of three lectures delivered by Prof. Marco Morandotti (Politecnico di Torino), each lasting approximately 45 minutes, followed by discussions with participants from several institutions.

The aim of the seminar was to introduce an analytical framework for the study of multi-agent systems in which the evolution of agents is influenced not only by their spatial configuration but also by strategic or categorical information represented by labels. Such models arise in a variety of contexts, including collective behavior, evolutionary game theory, and social or biological dynamics. The seminar provided a rigorous introduction to these models and discussed how they can be analyzed in the limit of a large number of agents.

Scientific Content

The lectures focused on mathematical models describing the dynamics of a large number of interacting agents whose state is characterized by both a spatial variable and a label variable. The spatial component typically evolves in a Euclidean space, while the label variable represents strategies or group affiliations and is modeled by a probability measure. The resulting framework allows one to capture both the spatial distribution of agents and the evolution of their strategies.

A central theme of the seminar was the passage from a discrete description of interacting agents to a continuum description in terms of partial differential equations. When the number of agents becomes very large, the empirical distribution of agents converges to a density that evolves according to a mean-field equation. In deterministic settings, the limiting equation typically takes the form of a continuity equation, while in the presence of stochastic effects it becomes a Fokker–Planck equation. This approach provides a powerful tool for analyzing complex systems, since it replaces the tracking of many individual agents with the study of a single macroscopic quantity.

A case study presented during the seminar concerned evolutionary game theory, and in particular the spatially inhomogeneous replicator equation. The classical replicator equation describes the evolution of strategies in a population under selection dynamics. By incorporating spatial dependence, one obtains a model in which both the spatial distribution of agents and their strategic composition evolve over time. The seminar discussed how such equations arise naturally from multi-agent models through mean-field limits.

Lecture titles and abstracts

Lecture 1: **Introduction**. Setting of the problem, and standing hypotheses on the velocity vector fields for the agent variable (a point in the Euclidean space) and for the label variable (a probability measure). Typical dynamics included in this setting: deterministic vs stochastic, through the addition of the Brownian motion.

Lecture 2: **A case study from evolutionary game theory**. The spatially inhomogeneous replicator equation. Introduction to the classical replicator equation; addition of the spatial dependence; mean-field limit.

Lecture 3: **Derivation of the replicator equation from the Moran process**. Introduction to the Moran process; extension to multiple strategies; limit to the replicator equation in the weak selection regime.

References:
Ambrosio, Fornasier, Morandotti, and Savaré: *Spatially Inhomogeneous Evolutionary Games*, CPAM, 2021.
Hofbauer and Sigmund: *Evolutionary games and population dynamics.* Cambridge University Press 1998.
Morandotti and Orlando: *Replicator dynamics as the large population limit of a discrete Moran process in the weak selection regime: A proof via Eulerian specification.* ESAIM:COCV, 2025.
Nowak: *Evolutionary Dynamics: Exploring the Equations of Life.* Belknap Press (2006)

Outcomes

The seminar gathered researchers interested on multi-agent systems, mean-field limits, and evolutionary dynamics, highlighting the connections between microscopic stochastic models and macroscopic continuum descriptions.

The event also provided valuable exposure for graduate students and early-career researchers to current research topics at the interface of applied analysis, probability, and mathematical modeling.

Overall, the seminar successfully achieved its objective of presenting recent developments in the mathematical analysis of multi-agent systems, and it stimulated fruitful discussions that may lead to future collaborations.

開催日：2025/04/28〜2025/04/28

Finite Element solver for kinematically incompatible non-simply connected Föppl-von Kármán plates｜2025a033

カテゴリー：イベント　タグ：一般研究 短期共同研究

開催概要

- **開催方法**：対面開催
- **開催場所**：九州大学 伊都キャンパス ウエスト1号館 C棟 5階 中講義室（W1-C-512）
- **主要言語**：英語
- **主催**：九州大学マス・フォア・インダストリ研究所
- **種別・種目**：一般研究-短期共同研究
- **研究計画題目**：Finite Element solver for kinematically incompatible non-simply connected Föppl-von Kármán plates
- **研究代表者**：Fabbrini Edoardo（九州大学　大学院数理学研究院・博士課程）
- **研究実施期間**：2025年4月21日(月) 〜 2025年5月3日(土)
- **公開期間**：2025年4月28日(月)
- **研究計画詳細**：https://joint2.imi.kyushu-u.ac.jp/research_chooses/view/2025a033

プログラム

4月28日（月）

13:00-14:00

Fabbrini Edoardo（九州大学, Graduate School of Mathematics）
Kinematically incompatible von-Kármán plates: analysis and numerics.

受付中

開催日：2026/02/24〜2026/02/26

Finite Element solver for kinematically incompatible non-simply connected Föppl-von Kármán plates②｜2025a033

カテゴリー：イベント　　タグ：一般研究 短期共同研究

開催概要

- **開催方法**：オンライン開催
- **主要言語**：英語
- **主催**：九州大学マス・フォア・インダストリ研究所
- **種別・種目**： 一般研究-短期共同研究
- **研究計画題目**：Finite Element solver for kinematically incompatible non-simply connected Föppl-von Kármán plates
- **研究代表者**：Fabbrini Edoardo（京都大学）
- **研究実施期間**：2025年4月21日(月) ～ 2025年5月3日(土),
  2026年2月24日(火) ～ 2026年2月26日(木)
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プログラム

2月24日（火）,25（水）,26日（木）

15：00-16：00(45-minute lecture followed by 15 minutes of Q&A)

Marco MORANDOTTI（Politecnico di Torino）
Multi-agent dynamics with labels and their continuum limit

Three 45-minute lessons.

Many complex systems arising from the modeling of natural or artificial phenomena, even social sciences, involve many particles, or agents whose evolution might be affected by the choice of a strategy, or as a consequence of belonging to a certain subgroup of agents, identified by a label. Studying these systems can help us understand these phenomena, make prediction and even account for rather general degrees of stochasticity.
In this course, we will set up a powerful analytical framework to study these systems and see a few examples. This framework will allow us to perform the limit as the number of agents diverges to infinity, thus passing from the discrete, Lagrangian description to that of the density of agents with strategies. The advantage of this approach is to describing the evolution of a single quantity through a PDE, rather than following those of the individual agents. The typical limit PDE is a continuity equation for deterministic dynamics and a Fokker—Planck equation in case stochasticity is accounted for.

The plan of the classes is the following:

1. Introduction, setting of the problem, and standing hypotheses on the velocity vector fields for the agent variable (a point in the Euclidean space) and for the label variable (a probability measure). Typical dynamics included in this setting: deterministic vs stochastic, through the addition of the Brownian motion.
2. A case study from evolutionary game theory: the spatially inhomogeneous replicator equation. Introduction to the classical replicator equation; addition of the spatial dependence; mean-field limit.
3. Derivation of the replicator equation from the Moran process. Introduction to the Moran process; extension to multiple strategies; limit to the replicator equation in the weak selection regime.

References
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Hofbauer and Sigmund: Evolutionary games and population dynamics. Cambridge University Press 1998.
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